An Efficient Framework for Computing Structure-And Semantics-Preserving Community in a Heterogeneous Multilayer Network

Abstract—

Multilayer networks or MLNs (also called multiplexes or network of networks) are being used extensively for modeling and analysis of data sets with multiple entity and feature types and associated relationships. Although the concept of community is widely-used for aggregate analysis, a structure- and semantics-preserving definition for it is lacking for MLNs. Retention of original *MLN structure* and *entity relationships* is important for detailed drill-down analysis. In addition, efficient computation is also critical for large number of analysis.

In this paper, we introduce a structure-preserving community definition for MLNs as well as a framework for its efficient computation using the decoupling approach. The proposed decoupling approach combines communities from individual layers to form a *serial k-community* for connected k layers in a MLN. We propose a new algorithm for pairing communities across layers and introduce several weight metrics for composing communities from two layers using participating community characteristics. In addition to the definition, our proposed approach has a number of desired characteristics. It: i) leverages extant single graph community detection algorithms, ii) introduces several weight metrics that are customized for the community concept, iii) is a new algorithm for pairing communities using bipartite graphs, and iv) experimentally validates the community computation and its efficiency on widely-used IMDb and DBLP data sets.

Index Terms—**Heterogeneous Multilayer Networks; Bipartite Graphs; Community Definition and Detection; Decoupling-Based Composition**

I. Motivation

As data sets become more complex in terms of entity and feature types, the approaches needed for their modeling and analysis also warrant extensions or new alternatives to match the data set complexity. With the advent of social networks and large data sets, we have already seen a surge in the use of graph-based modeling along with a renewed interest in concepts, such as community and hubs used for their analysis.

Informally, MLNs[1] are *layers of networks* where each layer is a simple graph and captures the semantics of an attribute (or feature) of an entity type using an edge to represent that relationship. The layers can also be connected. If each layer of a MLN has the `same set of entities of the same type`, it is termed a homogeneous MLN (or HoMLN.) For a HoMLN, intra-layer edges are shown explicitly and inter-layer edges are implicit (and not shown.) If `the set and types of entities are different for each layer`, then relationships of entities across layers are also shown using explicit inter-layer edges. This distinguishes a heterogeneous MLN (or HeMLN) from the previous one.

A. Structure- and Semantics-Preservation

For a simple graph, a community preserves its structure in terms of node/edge labels and relationships. Preserving the structure of a community of a MLN (especially HeMLN) entails preserving their multilayer network structure and preserving semantics includes preserving node/edge types, labels, and importantly inter-layer relationships. In other words, each community should be a heterogeneous MLN in its own right! Current approaches, such as using the MLN as a whole [2], type-independent [3], and projection-based [4], [5], do not accomplish this as they aggregate (or collapse) layers into a simple graph in different ways. Importantly, aggregation approaches are likely to result in some information loss [1], distortion of properties [1], or hide the effect of different entity types and/or different intra- or inter-layer relationships as elaborated in [6]. Structure-preservation is critical for understanding a HeMLN community and for drill-down or detailed analysis of communities.

Without structure- and semantics-preservation, it is not possible to understand the result of the analysis as mapping to original node labels and relationships is extremely difficult (or even not possible) when more than several layers are involved. We will demonstrate from our experimental results how easily we can drill down to see patterns in terms of original labels.

B. Decoupling Approach For Efficiency

Decoupling as proposed in this paper is the equivalent of "divide and conquer" for MLNs. Research on modeling a data set as a MLN *and* computing on the whole MLN has not addressed efficiency issues. As with divide and conquer, decoupling requires partitioning (which comes from the MLN structure) and a way to compose partial (or intermediate) results. This paper uses a customized bipartite graph match as the composition function (referred to as Θ, in this paper) leading to efficient community detection on MLNs. The contributions of this paper are:

- Definition of structure- and semantics-preserving k-community for a HeMLN (Section IV),

[1] The terminology used for variants of multilayer networks varies drastically in the literature and many a times is not even consistent with one another. For clarification, please refer to [1] which provides an excellent comparison of terminology used in the literature, their differences, and usages clearly.

- Identification of a composition function and formalizing decoupling-based approach for k-community detection with an algorithm (Section V),
- Mapping of detailed analysis requirements of the data set using the k-community and weight choices (Section VII),
- A new bipartite match algorithm (termed Maximum Weighted Bipartite Coupling or MWBC) for composing layers and identification of useful weight metrics and their uniqueness (Section VI), and
- Experimental analysis using the IMDb and DBLP data sets to establish the validity of the proposed approach along with performance analysis (Section VIII.)

The paper is organized as indicated above with related work in Section II and conclusions in Section IX.

II. Related Work

As the focus of this paper is community definition and its efficient detection in HeMLNs, we present relevant work on simple graphs and MLNs. The advantages of modeling using MLNs are discussed in [1], [7].

Community detection on a simple graph involves identifying groups of vertices that are more connected to each other than to other vertices in the network. Most of the work in the literature considers **single networks or simple graphs** where this objective is translated to optimizing network parameters such as modularity [8] or conductance [9]. As the combinatorial optimization of community detection is NP-complete [10], a large number of competitive approximation algorithms have been developed (see reviews in [11].) Algorithms for community detection have been developed for different types of input graphs including directed [12], edge-weighted [13], and dynamic networks [14]. However, to the best of our knowledge, there is no community definition and detection that include node and edge labels, node weights as well as graphs with self-loops and multiple edges between nodes[2]. Even the most popular community detection packages such as Infomap [15] or Louvain [16], do not accept non-simple graphs.

Recently, community detection algorithms have been extended to **HoMLNs** (see reviews [17], [18].) Algorithms based on matrix factorization, cluster expansion philosophy, Bayesian probabilistic models, regression, and spectral optimization of the modularity function based on the supra-adjacency representation have been developed. However, all these approaches *analyze a MLN either by aggregating all (or a subset of) layers of a HoMLN using Boolean and other operators or by considering the entire MLN as a whole*.

Majority of the work on analyzing HeMLN (reviewed in [5], [19]) focuses on developing meta-path based techniques for determining the similarity of objects [20], classification of objects [21], predicting the missing links [22], ranking/co-ranking [23] and recommendations [24]. An important aspect to be noted here is that most of them do not consider the intra-layer relationships and concentrate mainly on the bipartite graph formed by the inter-layer edges.

[2]This is in contrast to subgraph mining, search, and querying of graphs where non-simple or attributed graphs are widely used.

The type-independent [3] and projection-based [4] approaches used for HeMLNs *do not preserve the structure or semantics of the community.* The type independent approach collapses all layers into a simple graph keeping *all* nodes and edges (including inter-layer edges) sans their types and labels. The same is true for the projection-based approach as well. The presence of different sets of entities in each layer and the presence of intra-layer edges makes structure-preserving definition more challenging for HeMLNs and also warrants a novel composition technique (as proposed in this paper.) A few existing works have proposed techniques for generating clusters of entities [25], but they have only considered the inter-layer links and not the networks themselves. This paper hopes to fill the gap for a *structure- and semantics-preserving community*.

III. Definitions

A **graph** G is an ordered pair (V, E), where V is a set of vertices and E is a set of edges. An edge (u, v) is a 2-element subset of the set V. The two vertices that form an edge are said to be adjacent or neighbors. In this paper we only consider graphs that are undirected.

A **multilayer network**, $MLN(G, X)$, is defined by two sets of graphs: i) The set $G = \{G_1, G_2, \ldots, G_N\}$ contains graphs of N individual layers as defined above, where $G_i(V_i, E_i)$ is defined by a set of vertices, V_i and a set of edges, E_i. An edge $e(v, u) \in E_i$, connects vertices v and u, where $v, u \in V_i$ and ii) A set $X = \{X_{1,2}, X_{1,3}, \ldots, X_{N-1,N}\}$ consists of bipartite graphs. Each graph $X_{i,j}(V_i, V_j, L_{i,j})$ is defined by two sets of vertices V_i and V_j, and a set of edges (also called links or inter-layer edges) $L_{i,j}$, such that for every link $l(a, b) \in L_{i,j}$, $a \in V_i$ and $b \in V_j$, where V_i (V_j) is the vertex set of graph G_i (G_j.)

For a HeMLN, X is explicitly specified. Without loss of generality, we assume unique numbers for nodes across layers and disjoint sets of nodes across layers[3].

We propose a **decoupling approach for HeMLN community detection**. Our algorithm is defined for combining communities from two layers of a HeMLN using a composition function and is extended to k layers (by applying pair-wise composition repeatedly.) We define a *serial k-community* to be a multilayer community where communities from k distinct connected layers of a HeMLN are combined in a specified order.

Our proposed decoupling approach for finding HeMLN communities is as follows;

(i) First use the function Ψ (here community detection) to find communities in each of the layers individually,

(ii) for any two chosen layers, construct a bipartite graph using their communities as meta nodes and create meta edges

[3]Heterogeneous MLNs can also be defined with overlapping nodes across layers (see [1]) which is not considered in this paper.

[4]Technically, this should be expressed as $((\Psi(G_2)\ \Theta_{2,1}\ \Psi(G_1))\ \Theta_{2,3}\ \Psi(G_3).)$ However, we drop Ψ for simplicity. In fact, Θ with its subscripts is sufficient for our purpose due to pre-defined precedence (left-to-right) of Θ. We retain G for clarity of the expression. ω_e is a weight metric discussed in Section VI.

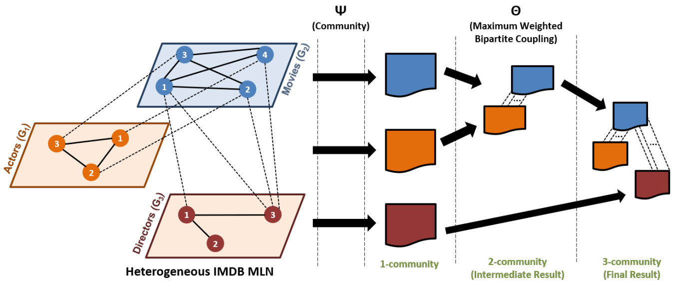


Fig. 1: Illustration of decoupling approach for computing a 3-community $((G_2\ \Theta_{2,1}\ G_1)\ \Theta_{2,3}\ G_3)$; ω_e[4]

that connect the meta nodes (using an appropriate element of X) and assign weight (ω). ω reflects the number of edges constituting a meta edge as well as properties of participating communities as discussed in Section VI, and

(iii) compose the partial results from each layer by representing each community as a meta node of the bipartite graph and using a function Θ which computes 2-community as pairs using the *weight information of edges in the bipartite graph.*)

Figure 1 illustrates the decoupling approach for specifying and computing a community of a larger size from partial results. It illustrates how a set of distinct communities from a layer is used for computing a 2-community (for 2 layers) and further a 3-community (for 3 layers) using partial results. 1-community is the set of communities generated for a layer (simple graph.)

We can define our problem statement as follows; *For a given data set with $\mathcal{F}$ different features and $\mathcal{T}$ entity types and a set of analysis objectives $\mathcal{A}$, model the data set using a HeMLN and determine the appropriate triad of Ψ, Θ, and ω, and a k-community expression for computing each objective.*

IV. Community Definition for a HeMLN

We first motivate the need for defining a structure- and semantics-preserving communities. For the IMDb data set, consider the HeMLM shown in Figure 1 and the analysis *"Find groups of actors for every director group such that the most versatile members interact?* Note that the actor and director layers can only compute groups of actors and directors, who act in or direct similar genre, respectively. The connection (or coupling) between directors and actors only come from inter-layer edges. It is only by preserving the structure of both the communities in actor and director as well as the inter-layer edges, can we compute the answer that indicates the semantics of which actor groups are paired with the director groups. The inter-layer edges preserve the relationships of individual actors and directors as well.

Clearly, multiple relationships can exist in such a collection of layers, such as co-acting, similar genres and who-directs-whom. An analysis requirement may also want to use *preferences* for community interactions. As an example, one may be interested in groups (or communities) where the *most important* actors and directors interact.

Our *definition of community for a HeMLN uses coupling of communities based on the connection strength (expressed as a weight) and is consistent with the simple graph definition of a community. Further, it also preserves the structure and semantics due to composition which is also shown to be efficient!*. Table I lists all notations used in the paper and their meaning for quick reference.

$G_i(V_i, E_i)$	Simple Graph for layer i
$X_{i,j}(V_i, V_j, L_{i,j})$	Bipartite graph of layers i and j
$MLN(G, X)$	Multilayer Network of layer graphs (set G) and Bipartite graphs (set X)
Ψ	Analysis function for G_i (community)
$\Theta_{i,j}$	Maximum Weighted Bipartite Coupling (MWBC) function
$CBG_{i,j}$	Community bipartite graph for G_i and G_j
U_i	Meta nodes for layer i 1-community
$L'_{i,j}$	Meta edges between U_i and U_j
c_i^m	m^{th} community of G_i
$v_i^{c^m}$, $e_i^{c^m}$	Vertices and Edges in community c_i^m
H_i^m	Hubs in c_i^m
$H_{i,j}^{m,n}$	Hubs in c_i^m connected to c_j^n
$x_{i,j}$	{Expanded(meta edge $< c_i^m,\ c_j^n >$)}
0 and ϕ	null community id and empty $x_{i,j}$
ω_e, ω_d, ω_h	Weight metrics for meta edges

TABLE I: Notations used in this paper

A. Formal Definition of Community in a HeMLN

A ***1-community*** is a set of communities of the simple graph corresponding to a layer.

A **community bipartite graph**[5] ***CBG***$_{i,j}(U_i,\ U_j,\ L'_{i,j})$ is defined between two disjoint and independent sets U_i and U_j. An element of U_i (U_j) is a 1-community id from G_i (G_j) that is represented as a single meta node. $L'_{i,j}$ is the set of meta edges between the nodes of U_i and U_j (or bipartite graph edges.) For any two meta nodes, a *single edge* is included in $L'_{i,j}$, if there is *an inter-layer edge* between any pair of nodes from the corresponding communities (acting as meta nodes in CBG) in layers G_i and G_j. Note that there may be many inter-layer edges between the communities from the two layers. Also note that U_i (U_j) need not include all community ids of G_i (G_j.) The strength (or weight) component of the meta edges is elaborated in Section VI.

A ***serial 2-community*** is defined on the community bipartite graph *CBG*$_{i,j}(U_i,\ U_j,\ L'_{i,j})$ corresponding to layers G_i and G_j A 2-community is a set of tuples each with a pair of elements $< c_i^m, c_j^n >$, where $c_i^m \in U_i$ and $c_j^n \in U_j$, that satisfy the *Maximum Weighted Bipartite Coupling (MWBC)* (composition function Θ defined in Section III) for the bipartite graph of U_i and U_j, along with the set of inter-layer edges between them. The pairing is done from left-to-right (hence it is *not commutative*) and a single community from the left layer can pair with *zero or several communities* from the right layer. That is, *one-to-many or many-to-one pairings*

[5]We defined the set X of bipartite graphs between layers of HeMLN in Section III. This is a different bipartite graph between two sets of nodes (termed meta nodes) from two distinct layers that correspond to communities in each layer. A single bipartite edge (termed meta edge) is drawn between distinct meta node pairs as defined.

are possible. The lower bound on the number of 2-community is $|U_i| - number\ of\ U_i\ nodes\ that\ have\ no\ outgoing\ edges.$

A ***serial k-community***[6] for k layers of a HeMLN is defined as the application of *serial 2-community* definition recursively to compose a k-community. The base case corresponds to applying the definition of 2-community for any two layers. The recursive case corresponds to applying 2-community composition for a t-community with another G_j.

For each recursive step, there are two cases for the 2-community under consideration: i) the U_i is from a layer G_i *already in the t-community* and the U_j is from a *new layer* G_j. This bipartite graph match is said to **extend** a t-community (t $<$ k) to a *(t+1)-community*, or ii) **both** U_i (U_j) from layers G_i (G_J) are *already in the t-community*. This bipartite graph match is said to **update** a t-community (t $<$ k), **not extend** it.

In both cases i) and ii) above, a number of outcomes are possible. Either a meta node from U_i: a) matches one or more meta nodes in U_j resulting in a (or many) **consistent match**, or b) does not match a meta node in U_j resulting in a **no match**, or c) matches a node in U_j that is not consistent with a previous match termed **inconsistent match**.

Structure preservation is accomplished by retaining, for each tuple of t-community, either a matching community id (or 0 if no match) and $x_{i,j}$ (or ϕ for empty set) representing inter-layer edges corresponding to the meta edge between the meta nodes (termed **expanded(meta edge).)** The *extend* and *update* carried out for each of the outcomes on the representation is listed in Table II. Note that due to multiple pairing of nodes during any composition, the number of tuples (or t-communities) may increase. Copying is used to deal with multiple pairings.

(G_{left}, G_{right}) **outcome**	**Effect on tuple *t***
`case (i) - one processed and one new layer`	
a) `consistent match`	**Copy & Extend** *t* with paired community id **and** $x_{i,j}$
b) `no match`	**Copy & Extend** *t* with 0 and ϕ
`case (ii) - both are processed layers`	
a) `consistent match`	**Copy & Update *t* only with** x
b) `no match`	**Copy & Update *t* only with** ϕ
c) `inconsistent match`	**Copy & Update *t* only with** ϕ

TABLE II: Cases and outcomes for MWBC (Algorithm 1)

A HeMLN can be viewed as a simple graph (termed HeMLN-graph) with each layer of a HeMLN being a node and the presence of inter-layer edges between layers denoted by an edge between corresponding nodes. Then, a k-community can be specified over any connected subgraph of the HeMLN-graph. Case i) above corresponds to a k-community of *an acyclic* subgraph of HeMLN-graph and case ii) to a k-community of a *cyclic* subgraph of the HeMLN-graph. For both, the number of compositions will be determined by the number of edges in the connected subgraph and can be more than the number of layers (or nodes). Also, for both cases, MWBC algorithm results in one of the 3 outcomes: a *consistent match* , *no match*, or an *inconsistent match (only for case (ii)* as shown in Table II.

B. Characteristics of k-community

The above definition when applied to a specification (such as the one shown in Figure 2) generates *progressively strong coupling between layers* (due to left-to-right precedence of Θ) using MWBC. *Thus, our definition of a k-community is characterized by dense connectivity within the layer (community definition) and strong coupling across layers (MWBC semantics.)*

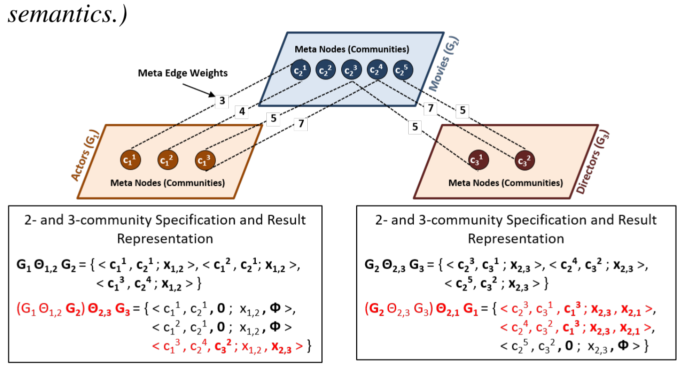


Fig. 2: Illustration of order dependence on a k-community

Space of Analysis Alternatives: Given a HeMLN with k layers, the number of possible k-community (or analysis space) is quite large. For a HeMLN-graph, the number of potential k-community is a function of the number of unique connected subgraphs of different sizes and the number of possible orderings for each such connected subgraph. With the inclusion of 3 weight metrics (see Section VI), it gets even larger. It is important to understand that each subgraph of a given size (equal to the number of edges in the connected subgraph) along with the ordering represents a *different* analysis of the data set and provides a different perspective thereby supporting a large space of analysis alternatives. Finally, the composition function Θ defined above is not commutative (due to left-to-right pairing) and also not associative[7]. Hence, for each k-community, the *order in which a k-community* is defined has a bearing on the result (semantics) obtained. In fact, the ordering is important as it differentiates one analysis from the other even for the same set of layers and inter-layer connections as elaborated in Section VII. Figure 2 shows clearly two 3-community for the same layers which are quite different!

Need for a new pairing algorithm: In a traditional bipartite graph (used for dating, hiring etc.), each node is a simple node. The goal is to find maximum number of matches (bipartite edges) such that no two matches share the same node. Hence, a node from one set is paired with at most one node from the other set. This has been extended to include weights for the edges without changing the pairing semantics [26]. On

[6]k represents the number of layers used for computing the community, not the number of compositions. The **"serial"** prefix used for defining a k-community indicates the order used (but can be arbitrary) in its specification. A k-community corresponds to a connected subgraph of k layers. Our definition assumes left-to-right precedence for the composition function Θ. It is possible to define a k-community with explicit precedence specification for Θ. Also, other definitions are possible that may be order agnostic. Finally, we drop the repetitive "serial" prefix henceforth as we only refer to a serial k-community in the rest of the paper.

[7]Due to the use of a subset of meta nodes rather than the entire 1-community during any recursive step.

the other hand, for maximal network flow algorithms [27], a source and a sink is assumed and weights have to be given from source to each node which is impractical in our case.

In contrast, each meta node in our case is a community representing a group of entities with additional characteristics. For a k-community to be meaningful, we need to associate weights with edges to capture not only the number of edges but also characteristics of the participating communities as well. To capture this, we discuss a number of alternatives for weights (termed weight metrics ω) in Section VI, derived from real-world scenarios.

For pairing nodes of the bipartite graph, since traditional approaches are not suited for our coupling, we propose a edge weight-based coupling which reflects the semantics of the community. Each node from the first set is paired with *zero, one or more nodes* from the second set solely based on the outgoing edge weights of that node. This is repeated for each node from the first set. *Most importantly, unlike current alternatives in the literature for community of a MLN, there is no information loss or distortion or hiding the effect of different entity types or relationships in our definition.*

V. HeMLN k-Community Detection

In this section, we first present a specification of a k-community and elaborate on a structure-preserving representation for the result. Then we present an algorithm for the maximum weighted bipartite coupling approach discussed earlier. We propose a number of meaningful ways in which we can consider the strength of the coupling for the MWBC approach by providing alternative weight metrics based on participating community characteristics to match with analysis objectives.

A. HeMLN k-Community Representation

Linearization of a HeMLN structure is done using an order of specification which is also used for computation. Although a k-community need to be specified as an expression involving Ψ and Θ, as indicated earlier, we drop Ψ for clarity. For the layers shown in Figure 2, a 3-community computation shown is for the specification $((G_2\ \Theta_{2,3}\ G_3)\ \Theta_{2,1}\ G_1)$. We can drop the parentheses as the precedence of Θ is assumed. However, we need the subscripts for Θ to disambiguate a k-community specification when a composition is done on the layers already used. If the layers $G_1\ and\ G_3$ were also connected with inter-layer edges in Figure 2, a 3-community involving a cycle can be specified as $G_1\ \Theta_{1,2}\ G_2\ \Theta_{2,3}\ G_3\ \Theta_{3,1}\ G_1$.

A k-community is represented as a set of tuples. Each tuple represents a distinct element of a k-community and includes an ordering of k community ids as items (a path, if you will, connecting community ids from different layers) and at least (k-1) expanded(meta edge) (i.e., $x_{i,j}$ elements.) This representation completely preserves the MLN structure along with semantics (labels) to reconstruct a HeMLN for any k-community. It is possible that there are multiple paths originating from the communities in the first layer of the expression due to one to many pairings. That is, a community in a layer can participate in more than one k-community tuple. All these paths need not remain total as the k-community computations progresses[8]. In summary, each k-community is a tuple with 2 distinct components. The first component is a comma-separated ordering of community ids (as items) from a distinct layer. The second component is also a comma-separated ordering of at least (k-1) $x_{i,j}$ (with each x having a different pair of subscripts.) Communities for x are uniquely identifiable from the subscripts. Figure 2 shows a number of 2- and 3-community results for the corresponding specifications. For the acyclic 3-community specification $G_1\ \Theta_{1,2}\ G_2\ \Theta_{2,3}\ G_3$, the element $< c_1^3,\ c_2^4,\ c_3^2\ ;\ x_{1,2},\ x_{2,3} >$ is the only total element as it does not include any ϕ, whereas, the other two elements, $< c_1^1,\ c_2^1,\ 0\ ;\ x_{1,2},\ \phi >$ and $< c_1^2,\ c_2^1,\ 0\ ;\ x_{1,2},\ \phi >$, are partial as both include one ϕ. Moreover, the partial elements share the c_2^1 showing that multiple paths can pass through the same meta node (community).

If G_1 and G_3 were connected, then for the 3-community specification $G_1\ \Theta_{1,2}\ G_2\ \Theta_{2,3}\ G_3\ \Theta_{3,1}\ G_1$ (involving a cycle), the total element shown in figure changes to $\{ < c_1^3,\ c_2^4,\ c_3^2\ ;\ x_{1,2},\ x_{2,3},\ x_{3,1} > \}$. In this case, the number of communities in each tuple is *k* (3 here) and the number of inter-layer edge sets is *at least (k-1)*. To generalize, an element of ***k-community*** for an arbitrary specification
$G_{n1}\ \Theta_{n1,n2}\ G_{n2}\ \Theta_{n2,n3}\ G_{n3}\ ...\ \Theta_{ni,nk}\ G_k$
will be represented as
$< c_{n1}^{m1},\ c_{n2}^{m2},\ ...,\ c_{nk}^{mk}\ ;\ x_{n1,n2},\ x_{n2,n3},\ ...,\ x_{ni,nk} >$, where some c's may be 0 and some x's may be ϕ.

B. Structure-Preserving Representation Benefits

1) Each element of a k-community can be further analyzed individually as the tuple contains all the information to reconstruct the HeMLN and drill down for details.
2) Total and partial elements of k-community provide important information about the result characteristics. A partial community shows a weak coupling of the complete community whereas a total element indicates strong coupling.
3) The resulting set can be ranked in several ways based on HeMLN community characteristics. For example, they can be ranked based on community size or density (or any other feature) as well as significance of the layer.

C. HeMLN k-Community Detection Algorithm

The MWBC algorithm identifies pairs of communities for the community bipartite graph input along with edge weights. Each node from the left set is paired with zero or more nodes from the other set. Either the highest edge weight pairs or all pairs with equal weight are output. This is important as the coupling strength is the same with multiple communities and all of them need to be in the result. This algorithm has been implemented using a single pass of the bipartite graph edges. Note that the number of communities from each layer is likely to be significantly less than the number of entity nodes in that layer. This translates to significantly less number of meta

[8]This is in contrast to the traditional pairing algorithms where any community can participate in *only one path of a k-community*.

edges in the bipartite graph compared to the total number of inter-layer edges between the layers.

Algorithm 1 HeMLN k-community Detection Algorithm

Require: -
INPUT: HeMLN, (G_{n1} $\Theta_{n1,n2}$ G_{n2} ... $\Theta_{ni,nk}$ G_{nk}), and a weight metric (wm).
OUTPUT: Set of distinct k-community tuples
Initialize: k=2, U_i = ϕ, U_j = ϕ, result$'$ = $\emptyset$
result $\leftarrow$ MWBC(G_{n1},G_{n2}, HeMLN, wm)
left, *right* $\leftarrow$ left and right subscripts of second Θ
while *left* $\neq$ null && *right* $\neq$ null **do**
  U_i $\leftarrow$ subset of 1-community($G_{left}, result$)
  U_j $\leftarrow$ subset of 1-community($G_{right}, result$)
  MP $\leftarrow$ MWBC(U_i, U_j, HeMLN, wm)
  `//a set of comm pairs` $< c^p_{left}, c^q_{right} >$
  for each tuple *t* $\in$ *result* **do**
    kflag = false
    if *both* c^x_{left} *and* c^y_{right} are part of *t* and $\in$ MP `[case ii (processed layer): consistent match]` **then**
      Update *a copy of t* with ($x_{left,\ right}$) and append to result$'$
    else if c^x_{left} is part of *t* and $\in$ MP and G_{right} layer has been processed `[case ii (processed layer): no and inconsistent match]` **then**
      Update *a copy of t* with ϕ and append to result$'$
    else if c^x_{left} is part of *t* and for each c^x_{left} $\in$ MP `[case i (new layer): consistent match]` **then**
      copy and Extend *t* with paired c^y_{right} $\in$ MP and $x_{left,\ right}$ and append to result$'$; kflag = true
    else if c^x_{left} is part of *t* and $\notin$ MP `[case i (new layer): no match]` **then**
      copy and Extend *t* with 0 (community id) and ϕ and append to result$'$; kflag = true
    end if
  end for
  left, *right* = next left, right subscripts of Θ or null
  if kflag k = k + 1; result = result$'$; result$'$ = $\emptyset$
end while

Algorithm 1 accepts a linearized specification of a k-community and computes the result as described earlier. The input is an *ordering of layers*, *composition function indicating the community bipartite graphs to be used* and the type of weight to be used. The output is a *set* whose *elements are tuples corresponding to distinct, single HeMLN k-community* for that specification. The size (i.e., number of tuples) of this set is determined by the pairs obtained during computation. The layers for any 2-community bipartite graph composition are identifiable from the input specification.

The algorithm iterates until there are no more compositions to be applied. The number of iterations is equal to the number of Θ in the input (corresponds to the number of inter-layer connections.) For each layer, we assume that its 1-community has been computed.

The bipartite graph for the base case and for each iteration is constructed for the participating layers (either one is new or both are from the t-community for some t) and MWBC algorithm is applied. The result obtained is used to either extend or update the tuples of the t-community and add new tuples as well. All cases are described in Table II. Note that the k-community size **k is incremented** only when a *new layer is composed (case i).)* For case ii) (cyclic k-community) **k is not incremented** when *both layers are part of the t-community*. When the algorithm terminates, we will have the set of tuples each corresponding to a single, distinct k-community for the given specification.

Figure 2 illustrates examples of 2- and 3-community results computed using the above algorithm. The figure also shows the two components of each tuple comprising of community id as items in the first component and the set of expanded(meta edge) or x as items for the second component. Further, it shows how the result of a 2-community is extended to form a 3-community. It also demonstrates the importance of order in which a k-community is defined. Further, it shows that the same result need be not obtained for different ordering (e.g., 3-community) *on the same layers*.

VI. Customizing MWBC

Algorithm 1 in Section V uses a bipartite graph match with a given weight metric. As we indicated earlier, there is an important difference between simple and **meta nodes/edges** that represent a **community of nodes/set of edges**. Without including the characteristics of meta nodes and edges for the match, we cannot argue that the pairing obtained represents analysis based on participating community characteristics. Hence, it is important to identify how qualitative community characteristics can be mapped quantitatively to a weight metric (that is, weight of the meta edge in a community bipartite graph) to influence the bipartite matching. Below, we propose three weight metrics and their intuition.

A. Number of Inter-Community Edges (ω_e)

This metric uses number of inter-community edges of participating communities as weight (normalized) for meta-edges. The intuition behind this metric is *maximum connectivity* (size of the community is to some extent factored into it) without including other community characteristics. This weight connotes *maximum interaction between two communities*.

B. Hub Participation (ω_h)

For many analysis, we are interested in knowing whether highly influential nodes within a community also interact across the community. This can be translated to the *participation of influential nodes within and across each participating community* for analysis. This can be modeled by using the notion of **hub participation** within a community and their interaction across layers. In this paper, we have used degree centrality for this metric to connote higher influence. Ratio of participating hubs from each community and the edge fraction are multiplied to compute ω_h. Formally,

For every $(u_i^m, u_k^n) \in L'_{i,k}$, where u_i^m and u_k^n are the meta nodes denoting the communities, c_i^m and c_k^n in the community bipartite graph, respectively, the weight,

$$\omega_h(u_i^m,\, u_k^n) = \frac{|H_{i,k}^{m,n}|}{|H_i^m|} * \frac{|x_{i,k}|}{|v_i^{c^m}|*|v_k^{c^n}|} * \frac{|H_{k,i}^{n,m}|}{|H_k^n|},$$

where $x_{i,k} = \bigcup \{(a,b) : a \in v_i^{c^m}, b \in v_k^{c^n},$ *and* $(a,b) \in L_{i,j}\}$; H_i^m *and* H_k^n *are set of hubs in* c_i^m *and* c_k^n*, respectively;* $H_{i,k}^{m,n}$ *is the set of hubs from* c_i^m *that are connected to* c_k^n*;* $H_{k,i}^{n,m}$ *is the set of hubs from* c_k^n *that are connected to* c_i^m *.*

C. Density and Edge Fraction (ω_d)

The intuition behind this metric is to bring participating community density which captures internal structure of a community. Clearly, *higher the densities and larger the edge fraction, the stronger is the interaction (or coupling) between two meta nodes (or communities.)* Since each of these three components (each being a fraction) increases the strength of the inter-layer coupling, they are multiplied to generate the weight of the meta edge. The domain of this weight will be $(0, 1]$. The weight computation formula is similar to the previous one.

Ideally, the alternatives for metrics should be independent of each other so they are useful for different analysis. Also, it is important that their computation be efficient (see Section VI-D.) We believe that the three metrics proposed satisfy the above and our experiments have confirmed them although not shown in this paper due to page constraints.

D. HeMLN k-community Detection Efficiency

For a given specification of a k-community, its detection has several cost components. Below, we summarize their individual complexity and cost.

1) **Cost of generating 1-community**: For each layer (or a subset of needed layers) this can be done in parallel bounding this **one-time cost** to the largest one (typically for a layer with maximum density.)
2) **Cost of computing meta edge weights**: For the proposed analysis metrics, part of them, again, are **one-time costs** and are calculated independently on the results of 1-community. The costs for ω_d and ω_h require a single pass of the communities using their node/edge details generated by the community detection algorithm.
3) **The recurring cost** (base case and each iteration): This includes the cost of generating the bipartite graph, computing the weight of each meta edge of the community bipartite graph for a given ω, and the MWBC algorithm cost. Only the edge fraction (or the maximum number of edges) and participating hubs need to be computed during each iteration. The cost of MWBC algorithm used in our experiments is O($|E|$), where E is the number of meta edges in the community bipartite graph. The bipartite graph is generated during the computation of weights for the meta edges. Luckily, in our community bipartite graph, the number of meta edges is **order of magnitude less** than the number of edges between layers. Also, the number of meta nodes is bound by the number of pairings in the previous iteration.

VII. ANALYSIS OF IMDb AND DBLP DATA SETS

We introduce the data sets and analysis objectives to formulate the k-community specification along with the choice of weight metric to apply the algorithm. This will clarify the usage of the three weight metrics for MWBC algorithm. The same data sets are used for experimental analysis.

DBLP data set [28]: The DBLP data set captures information about published research papers in conference/journal, year of publication and the authors. Most readers are familiar with this data set.

DBLP detailed Analysis Objectives

(A1) For each conference, which is the *most cohesive* group of authors who publish frequently?
2-community: P $\Theta_{P,Au}$ Au; ω_d

(A2) For the most popular collaborators from each conference, which are the 3-year period(s) when they were most active?
3-community: P $\Theta_{P,Au}$ Au $\Theta_{Au,Y}$ Y; ω_e

Based on the DBLP analysis requirements, three layers are modeled for the HeMLN. Layer *Au* connects any two authors (nodes) who have published at least three research papers together. Layer *P* connects research papers (nodes) that appear in the same conference. Layer *Y* connects two year nodes if they belong to same pre-defined period. The inter-layer edges depict *wrote-paper ($L_{Au,P}$), active-in-year ($L_{Au,Y}$) and published-in-year ($L_{P,Y}$)*. For this paper, we have chosen all papers that were published from 2001-2018 in top conferences. Six 3-year periods have been chosen: [2001-2003], [2004-2006], ..., [2016-2018].

IMDb data set [29]: The IMDb data set captures movies, TV episodes, actor, directors and other related information, such as rating.

IMDb Detailed Analysis Objectives

(A3) Based on similarity of genres, for each director group which are the actor groups whose *majority of the most versatile members interact*?
2-community: D $\Theta_{A,D}$ A; ω_h

(A4) For the *most popular* actor groups, for each movie rating class, find the director groups with which they have *maximum interaction* and who also make movies with similar ratings.
Cyclic 3-community: M $\Theta_{M,A}$ A $\Theta_{A,D}$ D $\Theta_{D,M}$ M; ω_e

For addressing the IMDb analysis requirements, three layers for the IMDb data set are formed as follows. Layer *A* and Layer *D* connect actors and directors who act-in or direct *similar genres frequently* (intra-layer edges), respectively. Layer *M* connects movies within the same rating range. The inter-layer edges depict *acts-in-a-movie* ($L_{A,M}$), *directs-movie* ($L_{D,M}$) *and directs-actor* ($L_{A,D}$). There are multiple ways of quantifying the similarity of actors and directors based on movie genres they have worked in. A vector was generated with the number of movies for each genre he/she has acted-in/directed. In order to consider the similarity with respect to *frequency of genres*, in layer *Genre*, two actors/directors are connected if the Pearsons' Correlation between their corresponding genre vectors is at least 0.9[9]. Moreover, 10 ranges are considered - [0-1), [1-2), ..., [9-10] for movie ratings.

For a specific analysis, the characteristics of the communities connected in the bipartite graph need to be used as meta edge weight to get desired coupling. For example, *maximum interaction* and *most popular* in (A2) and (A4), are interpreted as the number of edges between the participating communities. In contrast, interaction with cohesive groups as in (A1), is interpreted to include community density as well. Versatility is mapped to participation of hub nodes in each group as in (A3).

To compute a k-community, k needs to be identified. (A1) and (A3) require 2-community. (A2) requires 3-community (for 3 layers) with an acyclic specification (using only 2 edges) starting with the Paper layer as the analysis is from that perspective (each Layer P community corresponds to a conference). (A4) requires a cyclic 3-community using inter-layer relationships between all layers in a particular order. Note that the analysis objectives have been chosen carefully to cover the weights discussed in the paper. The limitation on the number of analysis objectives is purely due to space constraints.

VIII. Experimental Analysis

We would like to point out that the choice of data sets and sizes were mainly for demonstrating the versatility of analysis using the k-community detection and its efficiency as well as drill-down capability based on structure- and -semantics preservation. We are not trying to demonstrate scalability in this paper. Also, instead of presenting communities, we have chosen to show a few important drill-down results to showcase the structure- and semantics-preservation of our approach.

Experiment Setup: For DBLP HeMLN, research papers published from 2001-2018 in VLDB, SIGMOD, KDD, ICDM, DaWaK and DASFAA were chosen. For IMDb HeMLN, we extracted, for the top 500 actors, the movies they have worked in (7500+ movies with 4500+ directors). The actor set was repopulated with the co-actors from these movies, giving a total of 9000+ actors. For this set of actors, directors and movies, the HeMLN with 3 layers described in Section VII was built. Widely used Louvain method ([16]) was used to detect layer-wise 1-communities. The k-community detection algorithm 1 was implemented in Python version 3.6 and was executed on a quad-core 8^{th} generation Intel i5 processor Windows 10 machine with 8 GB RAM.

A. Analysis Results

Individual Layer Statistics: Table III shows the layer-wise statistics for IMDb HeMLN. 63 Actor (A) and 61 Director (D) communities based on similar genres are generated. Out of the 10 ranges (communities) in the movie (M) layer, most of the movies were rated in the range [6-7), while least popular rating was [1-2). No movie had a rating in the range [0-1).

	Actor	Director	Movie
#Nodes	9485	4510	7951
#Edges	996,527	250,845	8,777,618
#Communities (Size $>$ 1)	63	61	9
Avg. Community Size	148.5	73	883.4

TABLE III: IMDB HeMLN Statistics

Similarly, DBLP HeMLN statistics are shown in Table IV. 591 Author (Au) communities are generated based on co-authorship. 6 Paper (P) communities are formed by grouping papers published in same conference. KDD (2942) and DASFAA (583) have highest and least published papers, respectively. Out of 6 ranges of years (Y) selected, the maximum and minimum papers were published in 2016-2018 (1978) and 2001-2003 (1421), respectively.

	Author	Paper	Year
#Nodes	16,918	10,326	18
#Edges	2,483	12,044,080	18
#Communities (size $>$ 1)	591	6	6
Avg. Community Size	3.3	1721	3

TABLE IV: DBLP HeMLN Statistics

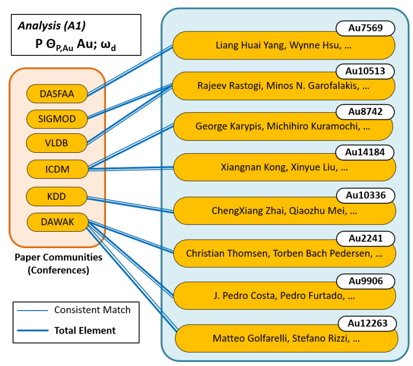


Fig. 3: (A1) Result: **7 Total Elements**[10]

(A1) Analysis: On applying MWBC on the CBG created with all Paper and Author communities, we obtained 7 total elements that correspond to the *most cohesive co-authors who also publish frequently in each conference* (shown in Figure 3 with list of few prominent authors.) ICDM and DaWaK have **multiple author communities** that are **equally important**. Researchers **George Karypis** and Michihiro Kuramochi **are members of one of the frequently publishing co-author groups (in the last 18 years) for ICDM (4 papers)**. Significance of this result is validated from the fact that George Karypis has been a recipient of **IEEE ICDM 10-Year Highest-Impact Paper Award (2010)**

[9]The choice of the coefficient is not arbitrary as it reflects relationship quality. The choice of this value can be based on how actors/directors are weighted against the genres. We have chosen 0.9 for connecting actors in their top genres.

[10]Louvain numbers all communities from 1 and we only consider communities having *at least two members* for this paper. The numbering used in the paper have layer name followed by the Louvain-generated community ID (e.g. A91, Au8742).

and IEEE ICDM Research Contributions Award (2017). Moreover, **multiple conferences can have same cohesive co-author groups**. For example, **co-authors Rajeev Rastogi and Minos N. Garofalakis are strongly associated with SIGMOD (7 papers) and VLDB (4 papers) in the past 18 years**[11].

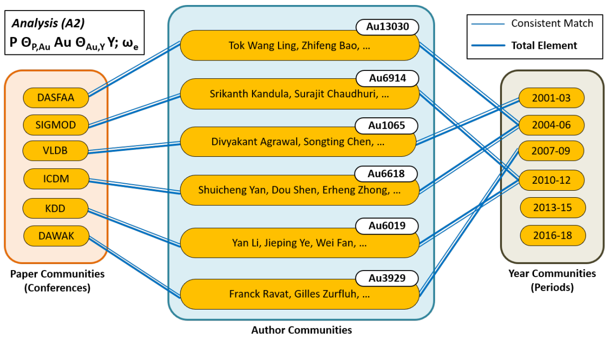


Fig. 4: (A2) Result: **6 Total Elements**

(A2) Analysis: For the required *acyclic 3-community* results, the *most popular author groups* for *each conference* are obtained by MWBC (first composition). The matched 6 author communities are carried forward to find the year periods in which they were *most active* (second composition). 6 total elements are obtained (path shown by **bold blue lines** in Figure 4.) Few prominent names have been shown in the Figure 4 based on citation count (from Google Scholar profiles.) Clearly, multiple co-author groups can be active in the same year for different conferences as seen from the results. **For SIGMOD, VLDB and ICDM the most popular researchers include Srikanth Kandula (15188 citations), Divyakant Agrawal (23727 citations) and Shuicheng Yan (52294 citations), respectively who have been active in different periods in the past 18 years**.

An interesting point to be noticed here is that none of the 6 author groups (*obtained from first composition*) had 2013-2015 and 2016-2018 as the most active periods. This is where the relevance of order comes which is derived from the analysis objectives.

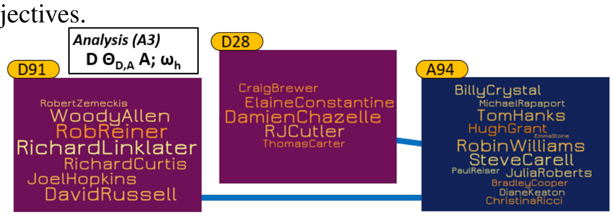
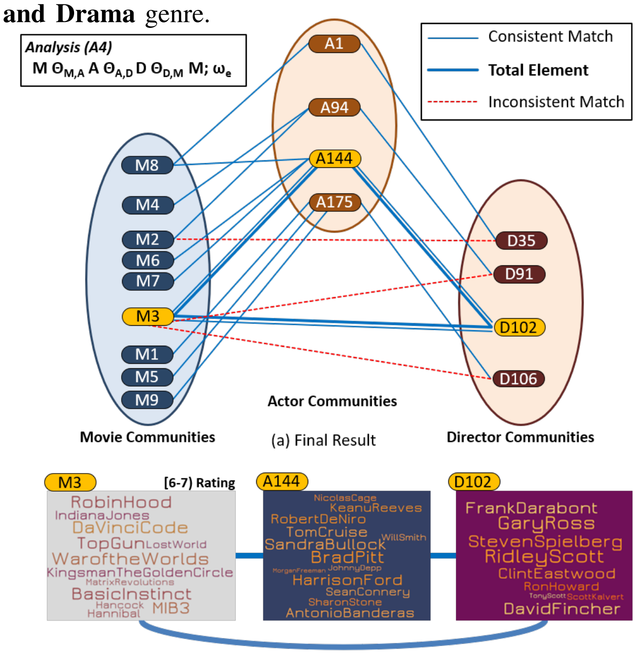


Fig. 5: Sample (A3) Result for *Romance, Comedy, Drama*

(A3) Results: 34 D-A (Director-Actor) similar genre-based community pairs were obtained, where *majority of most versatile members interact*. Intuitively, a group of directors that prominently makes movies in some genre (say, Drama, Action, Romance, ...) must pair up with the group(s) of actors who primarily act in similar kind of movies. Moreover, a *director group may work with multiple actor groups and vice-versa*. For example, in Figure 5, the sample result shows that the director groups, D28 and D91, with academy award winners like **Damien Chazelle and Woody Allen, respectively, pair up with the actor group with members like Diane Keaton, Emma Stone and Hugh Grant**. Members from these groups are primarily known for movies from the **Romance, Comedy and Drama** genre.



(b) Sample *Movies, Actors and Directors* from the Total Element

Fig. 6: (A4) Result: **1 Total, 9 Partial Elements**

(A4) Results: Here, the *most popular actor groups for each movie rating class are further coupled with directors*. These *director groups are coupled again with movies to check whether the director groups also have similar ratings*. Results of each successive pairing (there are 3) are shown in Figure 6 (a) using the same color notation. Coupling of movie and actor communities (first composition) results in 10 consistent matches. When the base case is extended to the director layer (second composition) using all director communities and the matched 4 actor communities, we got 4 consistent matches. The final composition to complete the cycle uses 4 director communities and 9 movie communities as left and right sets of community bipartite graph, respectively. **Only one consistent match is obtained to generate the total element (M3-A144-D102-M3) for the cyclic 3-community (bold blue triangle.)** The resulting total element is from the **Action, Drama genre** as can be seen from the sample members shown in Figure 6 (b). It is interesting to see 3 inconsistent matches (red broken lines) between the communities which clearly indicate that all couplings are not satisfied by these pairs. These result in 9 partial elements **The inconsistent matches also highlight the importance of mapping an analysis objective to a k-community specification for computation.** If a different order had been chosen (viz. director and actor layer as the base case), the result could have included the inconsistent matches.

[11]Weights at the layer level are not considered in this analysis. Hence, for an author (e.g., Jiawei Han) who has authored large *number of papers*, his co-authors are distributed among different co-author communities due to lack of weight and hence does not come out. This clearly demonstrates the need for weighted communities at the layer level to increase analysis space as has been shown with meta edge weights.

B. *Efficiency of Decoupled Approach*

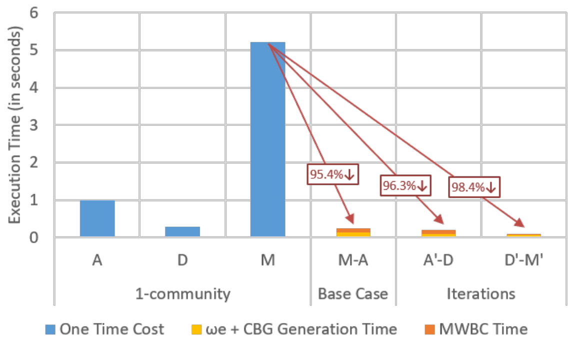


Fig. 7: Performance Results for cyclic 3-community in (A4)

The goal of the decoupling approach was to preserve the structure as well as improve the efficiency of k-community detection using the divide and conquer approach. We illustrate that with the largest k-community we have computed which uses 3 iterations (including the base case.) Figure 7 shows the execution time for the one-time and iterative costs discussed earlier for (A4). The difference in one-time 1-community cost for the 3 layers follow their density shown in Table III. We can also see how the iterative cost is insignificant as compared to the one time cost (by an order of magnitude.) Iteration cost includes creating the bipartite graph, computing ω_e for meta edges, and MWBC cost. **The cost of all iterations together (0.515 sec) is still almost *an order of magnitude less than the largest one-time cost* (5.21 sec for Movie layer.)** We have used this case as this subsumes all other cases. The **additional incremental cost for computing a k-community is extremely small validating the efficiency of decoupled approach**.

IX. Conclusions

In this paper, we have provided a structure- and semantics-preserving definition of a k-community for a HeMLN, its efficient computation, and drill-down of the results. We proposed a new bipartite-match based composition function that is better-suited for HeMLN Community composition. Finally, we used the proposed approach for demonstrating its analysis versatality using the IMDb and DBLP data sets.

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