



Association Rule Mining

Sharma Chakravarthy

Information Technology Laboratory (IT Lab)
Computer Science and Engineering Department
The University of Texas at Arlington, Arlington, TX

Email: sharma@cse.uta.edu URL: http://itlab.uta.edu/sharma



Association rules

- ➤ Capture co-occurrence of items / events
 - Not causality, which is to be inferred by a domain expert!
- ➤ Also called market basket analysis / link analysis
- ➤ Input: Transactions; each Ti is a set of items
- Problem: Find rules that can indicate good cooccurrences in the data set
- First paper appeared in Sigmod 1993 (Agarwal, Imielinksi, and Swami)



3



Motivation

- > Walmart has lots of data about point of sales
 - data on each user and the basket of items that has been bought during each visit
- Similarly, phone companies have information available about phone calls made at a particular time to a location
- Also, you have logs of what urls have been visited in each session and how much time has been spent at each url or session, what has been bought etc.
- How can the above information be leveraged for direct marketing, better placement of items on shelves etc.
 - In other words, for improving a business, i.e., deriving business intelligence (BI)!





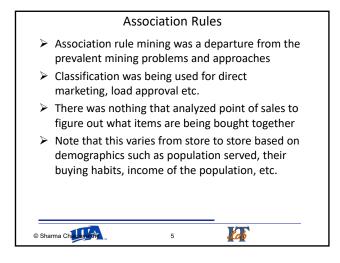
Terminology

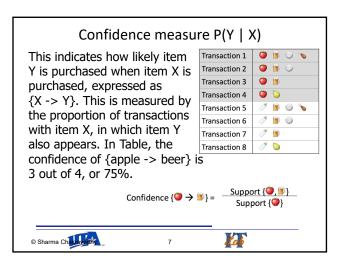
- ightharpoonup I = I₁, I₂,..., I_m set of items (sold) Large
- > T = a database of transactions ti very large
- ightharpoonup t[k] = 1 if t bought item I_k , t[k] = 0 otherwise
- ➤ An itemset is a (proper) subset of the number of items in a Tx ti
- ➤ let X be a subset of items in I
 - t satisfies X if for all items I_k in X, t[k] = 1

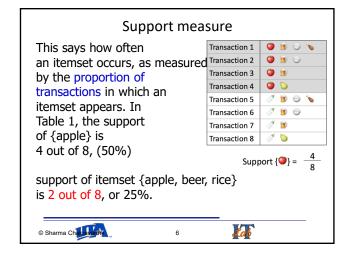


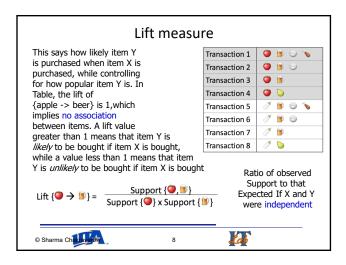


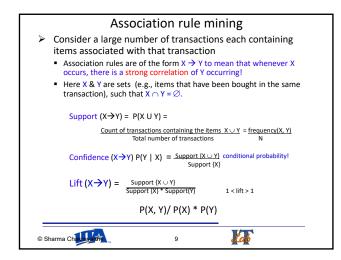
Kab

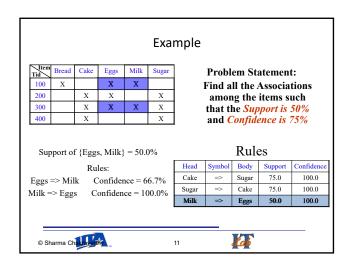












Association Rules: Details

- ➤ To discover associations, we assume that we have a set of transactions, each transaction being a list of items (e.g., list of books, items bought)
- Suppose A and B appear together in only 1% of the transactions but whenever A appears there is 80% chance that B also appears
- ➤ The 1% presence of A and B together is called the support of the rule and 80% is called the confidence of the rule (A → B)



Association rules

- ➤ Beer → diapers rule became a widely used example to illustrate that fathers watching super bowl (or sports) and taking care of babies shopped for these 2 items together!
- Other provoking examples tried to enhance the utility of this approach!

12



Lab

Association Rules

- Support indicates the frequency of the pattern. A minimum support (min_sup) is necessary if an association is going to be of some business value.
- ➤ A user might be interested in finding all associations which have x% support with y% confidence
- Also, all associations satisfying additional user constraints may be needed
- Also, associations need to be found efficiently from large data sets (need for real data sets and not samples)
- Confidence denotes the strength of the association. In addition, Lift can also be used



13



Number of itemsets

If there are n items, how many itemsets are possible?

$$_{n}C_{1} + _{n}C_{2} + ... + _{n}C_{n} = 2^{n}$$

- For n = 100, 2^{100} is approx. 1.27 * 10^{30}
- Typically, tens of thousands (Walmart sells more items than that)
- ➤ How many transactions? (basket or point of sales)
 - Typically in Millions
- The problem is to count frequency of itemsets satisfying min_sup and generate rules satisfying min_conf
 - So, why is this a problem?



15



Frequent Item

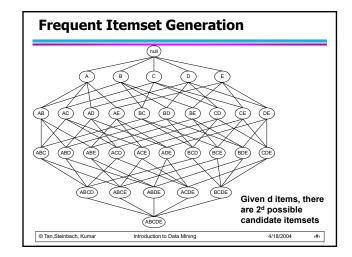
- ➤ A candidate itemset is any valid itemset
- ➤ A frequent Itemset is one that satisfies min_sup.
- Conceptually, finding association rules, is a simple two step approach:
 - Step 1 discover all frequent items that have support above the minimum support required
 - Step 2 Use the set of frequent items to generate all association rules that have high enough confidence

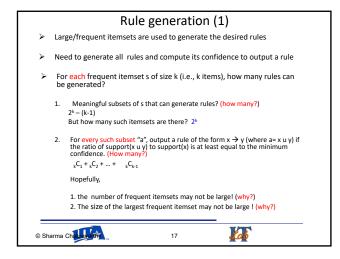
14

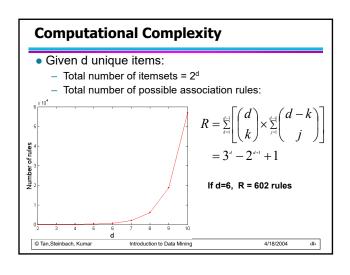
- Once we have frequent items (step 1) along with count, we can generate (enumerate) all rules to satisfy min_conf
- > Rule generation is a separate step!
- Our focus is on step 1

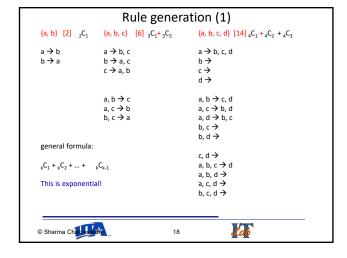


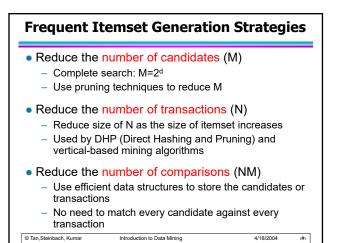
Kab

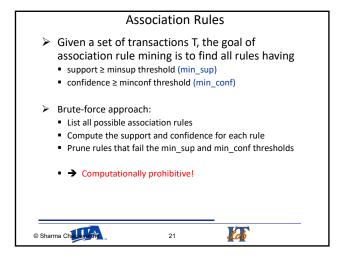


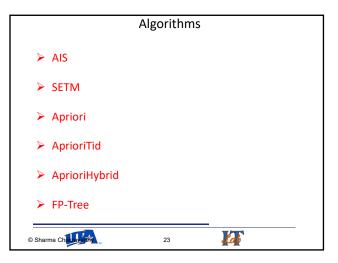


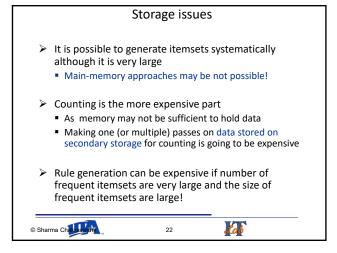


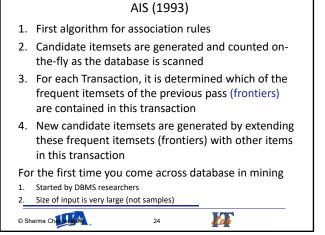












AIS Algorithm

- > The algorithm makes multiple passes over the database
- The *frontier* set for a pass consists of those items that are extended during the pass
- Candidate itemsets for the current pass are generated from the tuples in the database and the itemsets in the frontier set generated in the previous pass.
- Each itesmset has a counter to keep track of the number of transactions in which it appears (for min_sup checking)
- At the end of a pass, the support for a candidate itemset is compared with min_sup to determine if it is frequent
- It is also determined whether it should be added to frontier
- Algorithm terminates when frontier set is empty.



CSE 6331



AIS Frontier set

- For example, let I = {A, B, C, D, E, F} and assume that the items are ordered in
- Further assume that the frontier set contains only one itemset, AB.
- For the database tuple t = ABCDF, the following candidate itemsets are generated and predicted whether it is small or large (frequent):

ABC expected large: continue extending ABCD expected small: do not extend any further ABCF expected large: cannot be extended further ABD expected small: do not extend any further ABF expected large: cannot be extended further

- Statistical independence was used to estimate support for an itemset
- Product of prior probability and the database size is used for prediction
- Please look up the details in the paper (as part of my lectures)



CSE 6331



AIS Algorithm

- > All algorithms are iterative
- In the kth pass only those itemsets that contain exactly k items are computed/generated (candidate itemsets). And frequent itemsets of size k are identified.
- Having identified some itemsets in the kth pass, we need to identify in (k + 1)th pass only those itemsets that are 1-extensions (an itemset extended by exactly one item) of large itemsets found in the kth pass.
- If an itemset is small, its 1-extension is also going to be small (or not large or frequent). Thus, the frontier set for the next pass is set to candidate itemsets determined large (frequent) in the current pass, and only 1-extensions of a frontier itemset are generated and measured during a pass.
- Notion of expected to be large is introduced
- Remember, apriori property was not yet established!





CSE 6331



AIS Disadvantages

- > In AIS candidate sets were generated on the fly. New candidate item sets were generated by extending the large item sets that were generated in the previous pass with other items in the transaction.
- Frontier set generates more candidate itemsets than needed! There is pruning but it is approximate!
- Larger number of candidate itemsets were generated as compared to the frequent itemsets

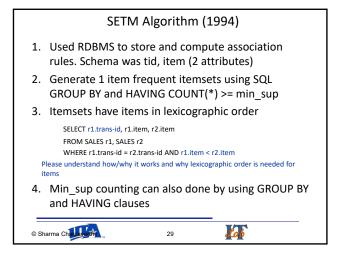
This algorithm uses some heuristics (prediction) so results may differ from ground truth

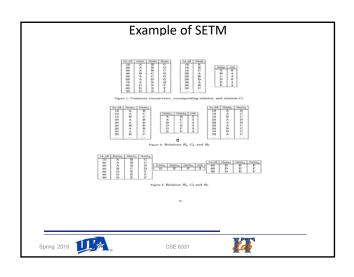
But, No false positives are generated! Some true positives may not be generated



28







SETM Algorithm Uses RDBMS to store and compute association rules. Schema is tid, item (2 attributes) Uses lexicographic order for correctness Uses sort-merge join (for efficiency, not correctness) sort R₁ on item; C1 := generate counts from R_1 ; repeat sort R_{k-1} on trans-id, item, . . . , item $_{k-1}$; $R_{k}' := merge\text{-scan } R_{k-1'} R_{i'} \text{ // sort-merge join was used}$ sort R_k' on $item_k$, ..., $item_k$; $c_k := generate counts from R_k';$ $R_k := filter R_k'$ to retain supported patterns; $until \ R_k = \{\}$ © Sharma Chakravarthy Lab 30

SETM discussion Candidate items are generated on-the-fly as the database is scanned (using sort-merge join), but counted at the end of the pass. New candidate items are generated in the same way as in AIS algorithm (which does not use a DBMS), but TID of the generating Tx is saved with candidate itemset as part of the relation At the end of the pass, the support count is computed using GROUP BY and non-frequent itemsets are filtered

SETM

- Showed that association rule mining can be done using SQL and RDBMS
- Sort-merge was used for speed up
- Specialized black-boxes were avoided
- Main memory is not a limitation any more
- ➤ Buffer management is handled by DBMS
- Query optimization is handled by DBMS
- Has the same limitations of AIS generates too many candidate itemsets – most of which does not become frequent!



31



Frequent Itemset Generation Strategies

- > Reduce the number of candidates (M)
 - Complete search: M=2^d
 - Use pruning techniques to reduce M
- > Reduce the number of transactions (N)
 - Reduce size of N as the size of itemset increases
 - Used by DHP (direct hashing and pruning) and verticalbased mining algorithms
- > Reduce the number of comparisons (NM)
 - Use efficient data structures to store the candidates or transactions
 - No need to match every candidate against every transaction





Apriori class of Algorithms

- Vldb94 paper presents 2 new algorithms: Apriori and AprioriTid
- These two outperform earlier algorithms (AIS and SFTM)
- The performance gap is shown to increase with the size, and ranges from a factor of 3 for small problems to more for larger problems.
- Also proposes Apriori Hybrid, which is a hybrid of the above 2 new algorithms and is found to have excellent scale up properties.



34



Comparison

- ➤ All the previous algorithms (AIS and SETM) that were used to determine all association rules were slow (relatively)
- ➤ This is due to the generation of a large number of sets which eventually turned out to be small (did not satisfy minimum support)
- In other words, they had lower support than the user defined minimum support.



36



Earlier Algorithms

➤ In case of the AIS and SETM, candidate sets were generated on the fly. New candidate item sets were generated by extending the large item sets that were generated in the previous pass with ALL items in the transaction.



0.7



The Apriori Algorithm

- The Apriori and AprioriTid algorithms generate the candidate itemsets to be counted in a pass by using only the itemsets found frequent in the previous pass – without considering ALL the transactions in the database.
- > The basic intuition is that all itemsets of a large/frequent itemset must be large/frequent
- Therefore, the candidate itemsets having k items can be generated by joining large itemsets having k-1 items, and deleting those that contain any subset that is not large.
- Apriori principle: if an itemset is frequent, then all its subsets must be frequent!



39



Earlier Algorithms

- Disadvantages
 - Algorithm makes passes over the data until the frontier set is empty; Frontier set is based on the number of itemsets that were expected to be small but turn out be large.
 - The number of combinations that are generated that turn out to be not large is considerably greater in these 2 algorithms.



38



Reducing Number of Candidates

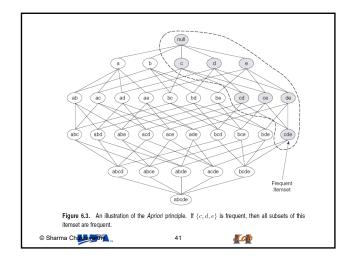
- > Apriori principle:
 - If an itemset is frequent, then all of its subsets must also be frequent
- > Apriori principle holds due to the following property of the support measure:

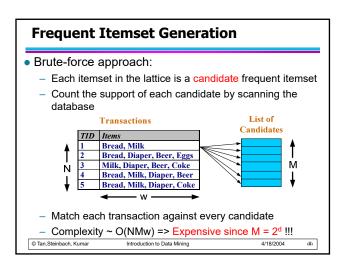
$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \ge s(Y)$$

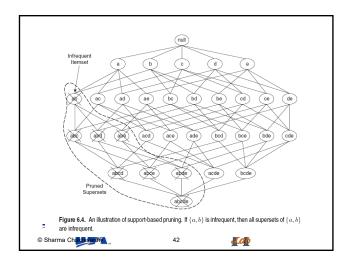
- Support of an itemset never exceeds the support of its subsets
- This is known as the anti-monotone property of support

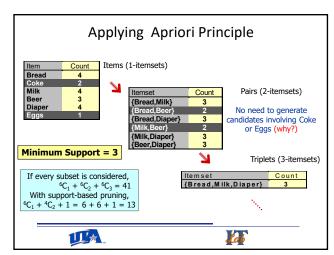




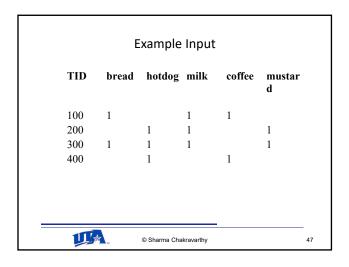


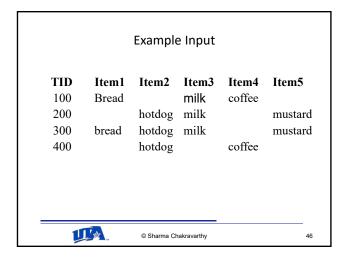


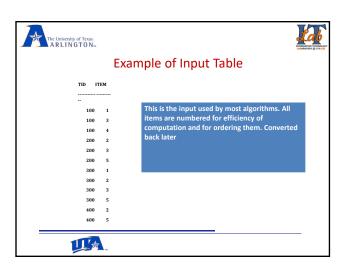


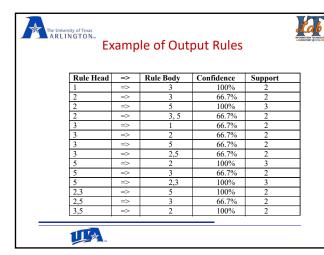


Frequent set discovery Level-wise search Closure property of frequent sets Apriori algorithm Hashtrees to hold candidate itemsets Techniques to reduce I/O and computation Sampling (guess and correct) Dynamic itemset counting









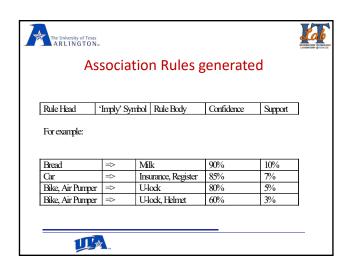
The Apriori steps

- Scan all transactions and find all 1-items that have support above min_sup. Let these be F1. (pass 1)
- Build item pairs from F1. This is the candidate set C2. Scan all transactions and find all frequent pairs in C2. Let this be F2 (support count) (pass 2)
- build sets of k items from Fk-1. This is set Ck.
- Prune Ck using the apriori principle!
 - Note this is not done in passes 1 and 2
- Scan all transactions and find all frequent sets in Ck. Let this be Fk. (pass k)
- > Stop when Fk is empty for some k
- > Generate all rules to satisfy confidence.
- How many # of passes in total?
- K (does not include candidate set generation, pruning)



51





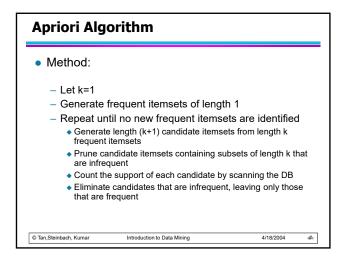
Pass 2 characteristics

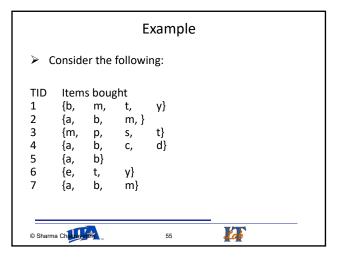
- > Scan all transactions and find all items that have transaction support above min_sup. Let these be F1.
- Build item pairs from F1. This is the candidate set C2.
 - no need for pruning! (why?)
 - C2 does NOT have any subsets that are not frequent!
- C2 is larger than C in any other pass! (why?)
- \triangleright $_{n}C_{3}$ is larger than any other combination! Confirm this!
- Scan all transactions and find all frequent pairs in C2. Let this be F2.
- General rule: build sets of k items from Fk-1. This is set Ck. Prune Ck. Scan all transactions and find all frequent sets (support count) in Ck. This be Fk.

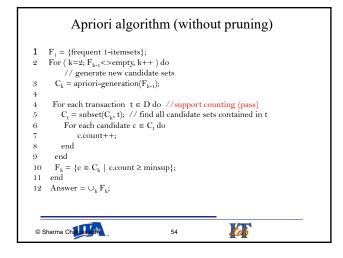


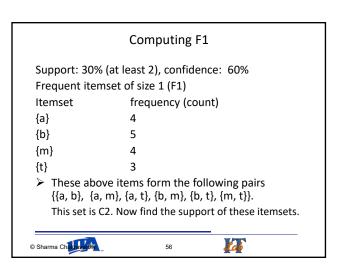
52











Computing F2

Item	count	Item	count
{a,b}	4	{b,m}	3
{a,m}	2	{b, t}	1
{a,t}	0	{m,t}	2

- > {a, b}, {a, m}, {b, m}, and {m, t} have 30% support
- > So, F2 is {a, b}, {a, m}, {b, m}, and (m, t}



Generating rules from F

- > All frequent itemsets are:
 - F1: {a}, {b}, {m}, {t} Union
 - F2: {a, b}, {a, m}, {b, m}, and (m, t} Union
 - F3: {a, b, m}
- No rules from F1 (why?)
- Rules from F2
 - $a \Rightarrow b$ has confidence of 3/3 = 100% $b \Rightarrow a$ has confidence of 3/5 = 60%

 - $b \Rightarrow m$ has confidence of 3/5 = 60%
 - $m \Rightarrow b$ has confidence of 3/3 = 100%
- Rules from F3 (compute confidence for these)
 - $a \rightarrow \{b, m\}, b \rightarrow \{a, m\}. M \rightarrow \{a, b\}$
 - $\{a, b\} \rightarrow \{m\}, \{a, m\} \rightarrow \{b\}, \{b, m\} \rightarrow \{a\}$





Computing F3

- > F2 is {a, b}, {a, m}, {b, m}, and (m, t}
- > C3 is {a, b, m}
- Support of {a, b, m} is 2. hence F3 is {a, b, m}
- > C4 is { {} }, however, empty! (why?)
- ➤ Need at least 2 F3 items to generate a C4 item!
- > The algorithm stops here. Note that we did not apply the pruning step. We will come to that later.





Apriori candidate generation

> The Apriori-generation function takes as argument F(k-1), the set of all frequent (k-1)-item sets. it returns a superset of the set of all frequent k-item sets. The function works as follows: First, in the join step, we join F(k-1) with F(k-1):

insert into C(k)

select p.item(1), p.item(2),... p.item(k-1), q.item(k-1)

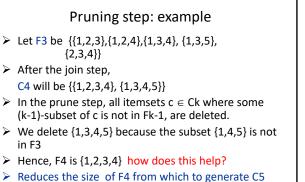
from F(k-1) as p, F(k-1) as q

where p.item(1) = q.item(1),...,p.item(k-2) = q.item(k-2),
p.item(k-1) < q.item(k-1) //pay attention to this condition</pre>

> Assumes lexicographic ordering of items



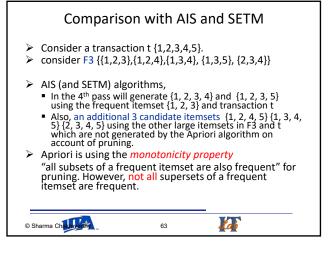


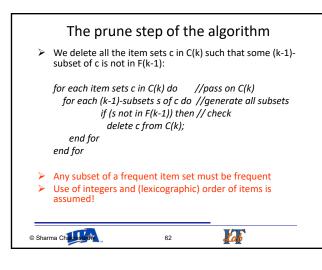


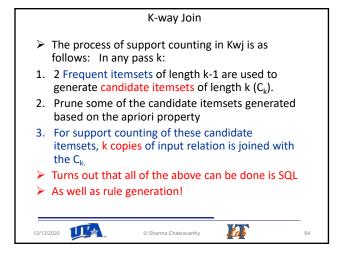
Lab

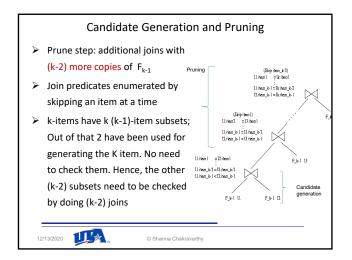
Still need to compute the support of F4 itemsets!

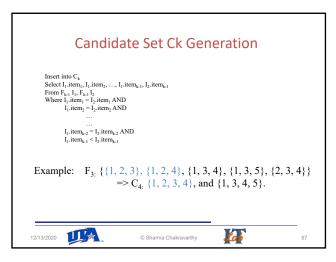
© Sharma Chakravarthy

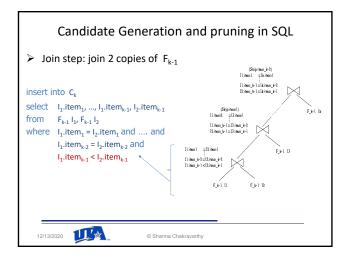


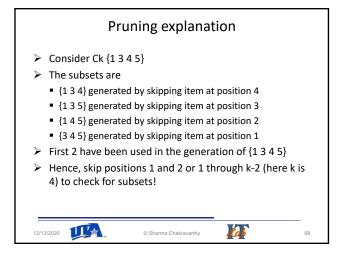


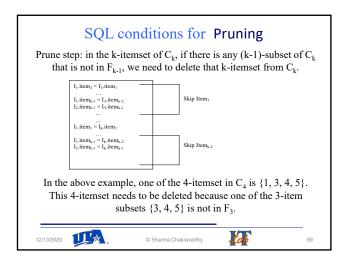


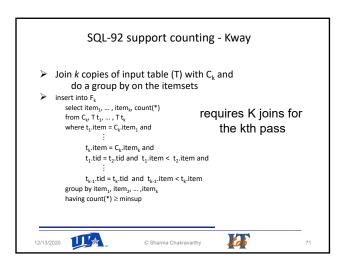


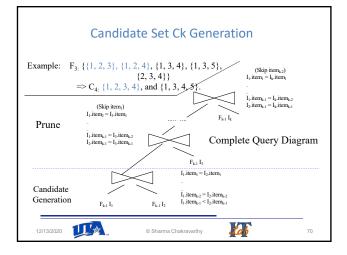


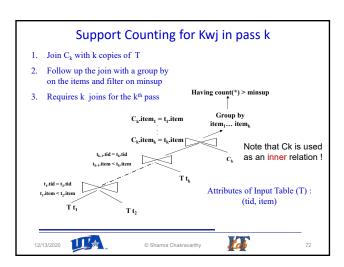


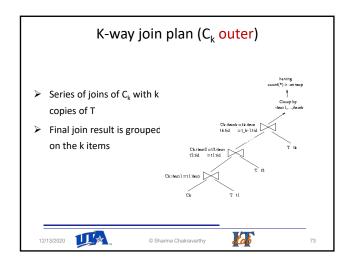


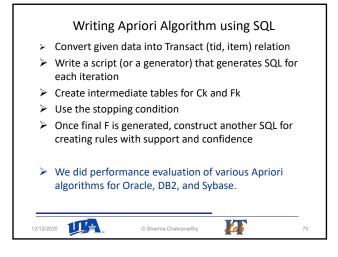


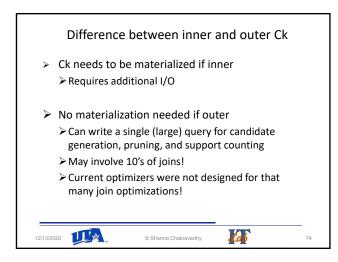


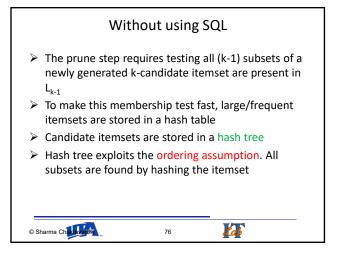


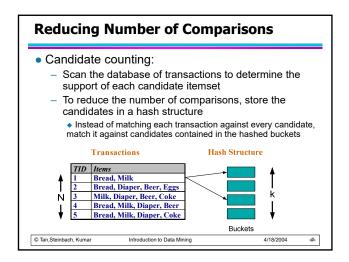


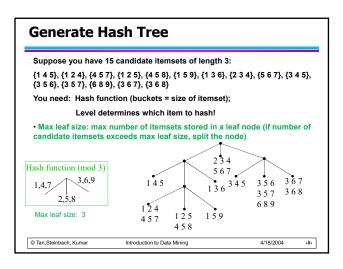


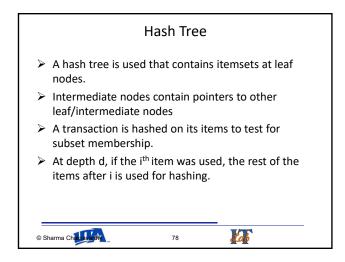


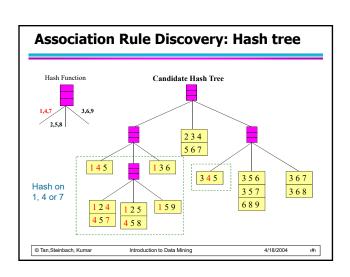


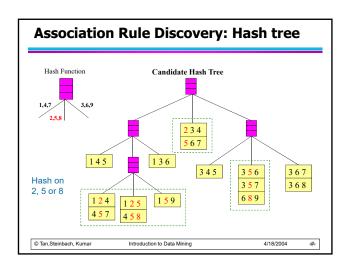


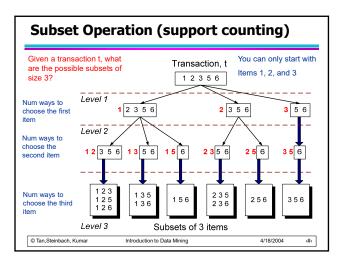


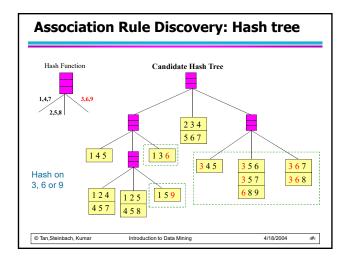


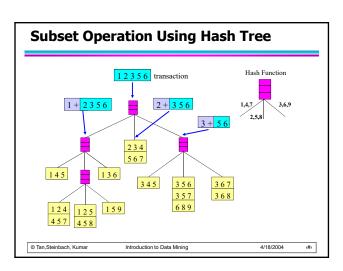


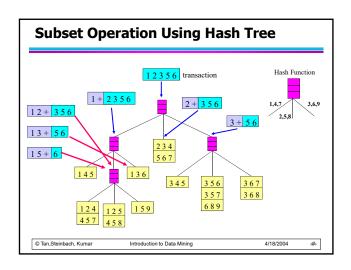








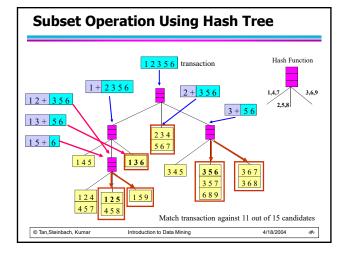


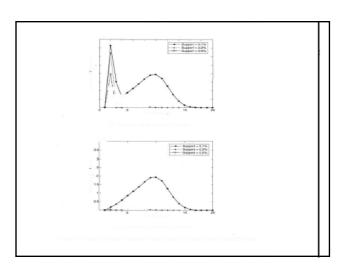


• Choice of minimum support threshold - lowering support threshold results in more frequent itemsets - this may increase number of candidates and max length of frequent itemsets

- Dimensionality (number of items) of the data set
 - more space is needed to store support count of each item
 - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
 - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
 - transaction width increases with denser data sets
 - This may i) increase max length of frequent itemsets and ii) traversals of hash tree (number of subsets in a transaction increases with its width)

© Tan,Steinbach, Kumar Introduction to Data Mining 4/18/2004 #>





Correctness

- > Ck is a superset of Fk
- > Ck is a superset of Fk by the way Ck is generated
- Subset pruning is based on the monotonicity property and every item pruned is guaranteed not be large
- ➤ Hence, Ck is always a superset of Fk



Impact of memory: Buffer management

- F_{k-1} fits in memory and C_k does not: generate as many C_k as possible, scan database and count support and write F_k to disk. Delete small itemsets. Repeat until all of F_k is generated for that pass.
 - # of passes on database (or transactions) is equal to the #of partitions of Ck due to memory limitation
 - Increases passes on the database!
- F_{k-1} does not fit in memory: externally sort F_{k-1}. Bring into memory F_{k-1} items in which the first k-2 items are the same. Generate Candidate itemsets. Scan data and generate Fk.
 - Unfortunately, pruning cannot be done (why?)



Buffer management

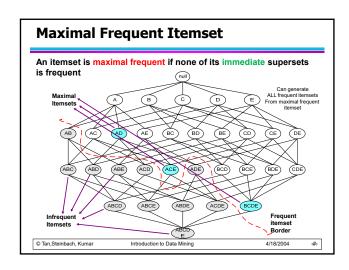
- ➤ In the candidate generation of pass k, we need storage for F_{k-1} and the candidate itemsets C_k
- In the counting phase of pass k, we need storage for C_k and at least one page to buffer the database transactions (C_t is a subset of C_k)
- Transactions are assumed be stored on the disk (whether in a database or not does not matter)

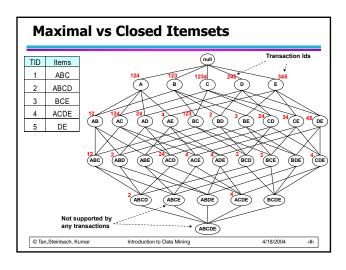


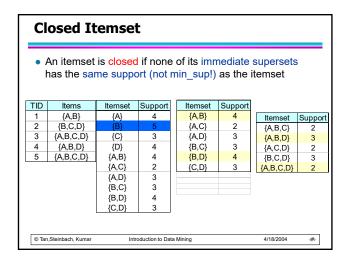
Effect of memory on pruning

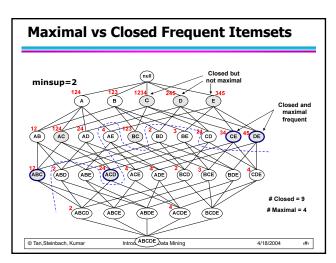
- > Let F3 be {{1,2,3},{1,2,4},{1,3,4}, {1,3,5}, {2,3,4}} sorted on firs 2 itemsets
- Suppose I can only load {{1,2,3},{1,2,4}, {1, 3, 4}} into memory
- C4 will be {{1,2,3,4}}
- ➤ To test all its subsets in F3, I need to check {2, 3, 4} is in F3 in addition to {1, 2, 3} and {1, 2, 4}
- However, {2, 3, 4} is NOT in memory. Hence cannot be checked!
- Only checking for {1, 2, 3} and {1, 2, 4} is not enough!

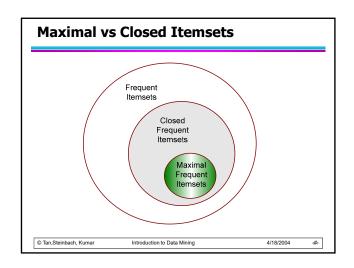


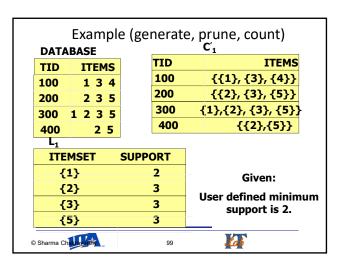












AprioriTid

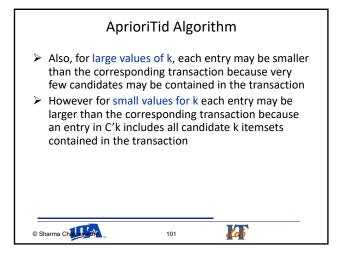
- It is similar to the Apriori Algorithm and uses Apriori-gen function to determine the candidate sets initially.
- But the basic difference is that for determining the support, the database <u>is not used</u> after the first pass.
- > Rather a set C'k is used for this purpose
- > Each member of C'k is of the form <TID, {Xk} > where Xk is potentially large/frequent k itemset present in the transaction with the identifier TID
- > C'1 corresponds to database D.

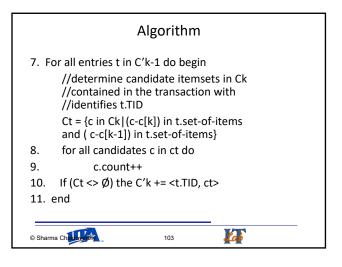


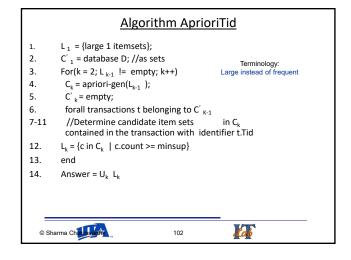
AprioriTid

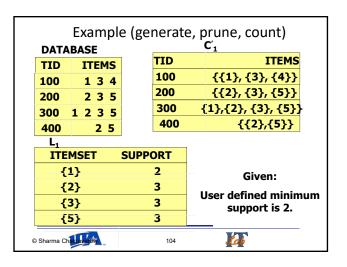
- If a transaction does not contain a candidate itemset, then C'k will not have any entry for this transaction
- Hence, for large values of k the number of entries in C'k may be much smaller than the number of transactions in the database (why?)
- Number of transactions in which this itemset exists decreases as the number of items in an itemset (k) increases!

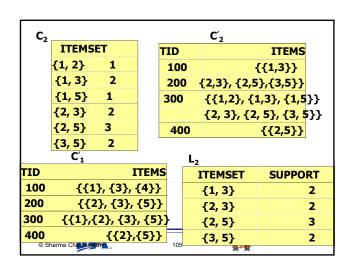


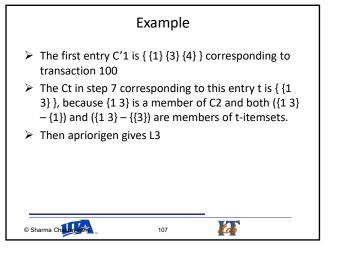


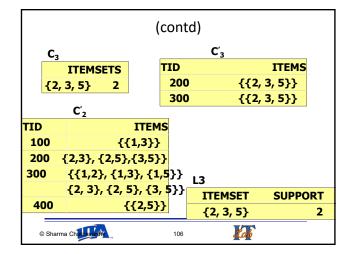


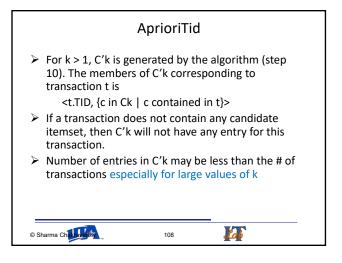


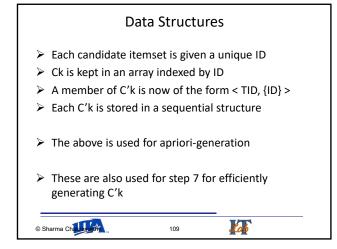


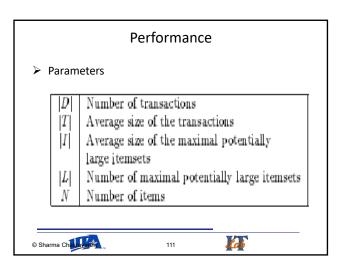


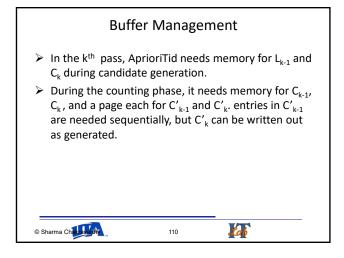


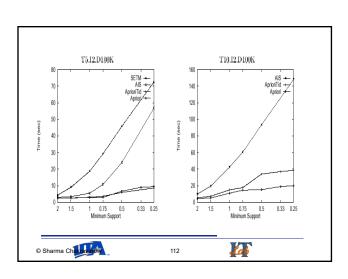


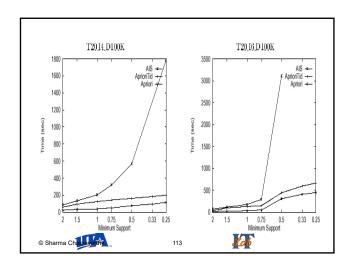


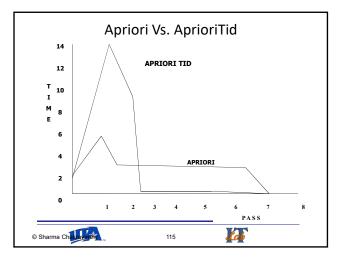












Disadvantages

- > Apriori Algorithm
 - For determining the support of the candidate sets the algorithm always looks into every transaction in the database. Hence it takes a longer time (more passes on data)
- > AprioriTid Algorithm
 - During initial passes the size of C'k is very large and is almost equivalent to the size of the database. Hence the time taken will be equal to that of Apriori. And also it might incur an additional cost if it cannot completely fit into the memory.

Lab



Algorithm Apriori Hybrid

- Idea: Not necessary to use the same algorithm in all passes over data.
- During the Initial Passes : Apriori Algorithm is used.
- During the Later Passes: AprioriTid Algorithm is used.
- Apriori Hybrid uses the Apriori in the initial passes and switches to AprioriTid when it expects that the set C'k at the end of the pass will fit into the memory.



Disadvantages of Apriori Hybrid

- > An extra cost is incurred for switching from Apriori to AprioriTid algorithm.
- Suppose at the end of K th pass we decide to switch from Apriori to AprioriTid. Then in the (k+1) pass, after having generated the candidate sets we also have to add the Tids to C'k+1
- If C'k remains large till the end then we do not get much benefits of using Apriori Hybrid Algorithm



Summary

- Data Mining is a tool box consisting of different tools designed for different purposes
- Choosing the appropriate tool is one of the difficult aspects of data mining
- Understanding the domain and matching the DM techniques for what one wants to do is another challenge
- Interpreting the results of the mining output is the third challenge
- Buying a DM system is much easier than using it effectively and improving business!



Conclusions

- The performance gap increased with the problem size and ranged from a factor of three for small problems to more than an order of magnitude for large problems.
- The algorithms presented in the paper have been implemented on several data repositories and were found to give consistent results.



